Essential Concepts

Thank you for coming to this experimental presentation.

The Executive Board of the CONCEPTS series asked me to deliver three short talks on “Essential Concepts”.

The idea is not to present new research results, but to illuminate some aspects of the theory which perhaps deserve more attention.

Today’s lecture is the most important one. You will probably be disappointed. That is why I am going to change the subject quickly.

Essential Concepts:

The hardest open problem of Formal Concept Analysis

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Ernst-Schröder–Zentrum Darmstadt

Montpellier, September 4, 2026

There's nothing left to discover?

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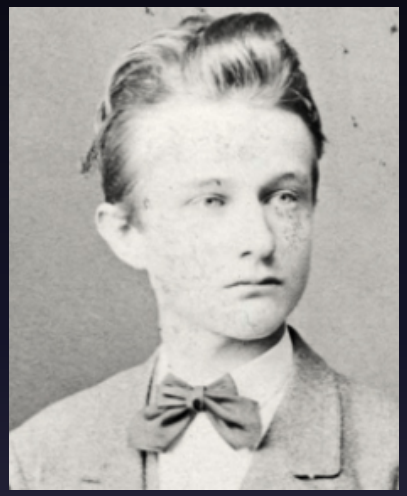

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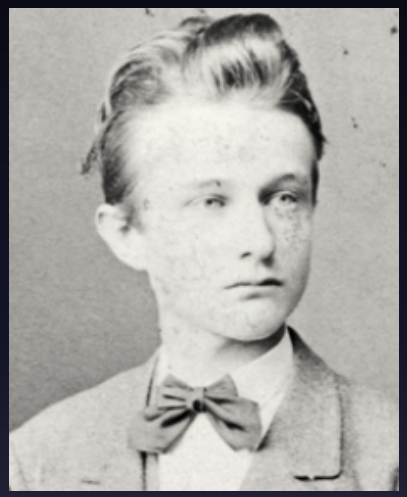

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Predictions like these are risky, but necessary. We should keep asking ourselves: What are the next important steps in the development of Conceptual Knowledge Structures?

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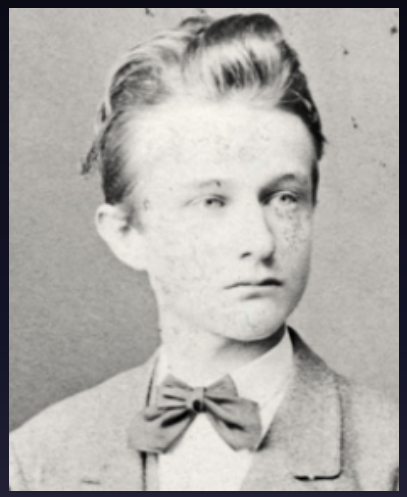

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What's left to discover in Formal Concept Analysis?

The hardest open problem

So here, in my opinion, is the most important unresolved problem of Formal Concept Analysis.

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Meaning Significance Importance Impact Usefulness
Benefit Value Effectiveness Applicability Validity

The hardest open problem

So here, in my opinion, is the most important unresolved problem of Formal Concept Analysis.

You are not going to like it:

What is the meaning of a formal concept?

It seems to me that, right now, Formal Concept Analysis' biggest weakness is that we do not convincingly explain why it is important.

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The philosopher Charles Sanders Peirce has some advice on this: seek out the “outward clash”! Expose yourselves to reality!

Deal with real world data, unvarnished and challenging. This will show you what your theory can do - and what it is still missing!

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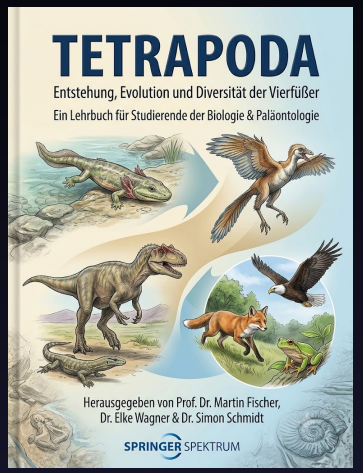
Imagine a biology student preparing for an exam, using a textbook which provides a comprehensive overview of all species of terrestrial vertebrates and their characteristics.



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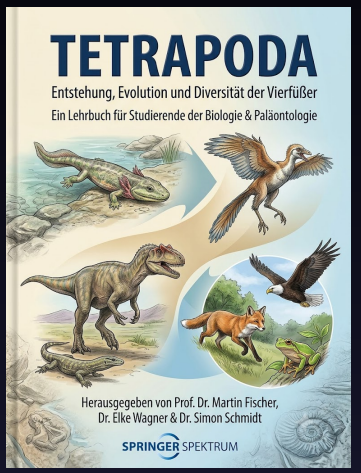

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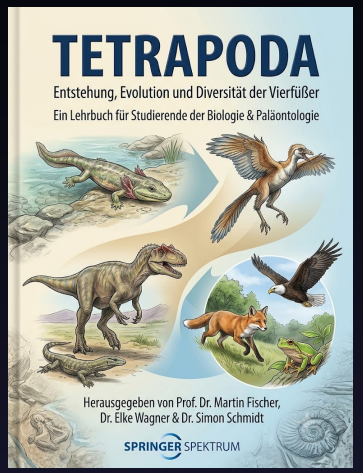

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Can we use FCA terminology to express what kind of knowledge is required?

What context, what concept?

With a little imagination, we can interpret the textbook as a formal context (G, M, I). The objects G are the species discussed in it, and the attributes M are the properties mentioned. We must hope that the authors provide sufficient information about the incidence I.

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"Snake" should then refer to a formal concept of this context. The extent includes all snake species listed in the book, and the intent consists of their common attributes, if mentioned.

Is the student expected to be able to present this formal concept? The answer is **no**!

The required knowledge is not a formal concept

It would be unrealistic to expect the candidate to know *all* the snake species described in the book.

It is realistic to expect that he/she is familiar with a *representative sample*.

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It is realistic to expect that he/she is familiar with a *representative sample*.

It is unrealistic to expect knowledge of *all* properties of snakes.

What is realistic is that the candidate can *define* what snakes are.

Formally . . .

Formally, an acceptable answer consists of stating

- a representative set E of examples of snakes,

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The required knowledge is of the form

$$(E, F) \quad \text{with} \quad E'' = F',$$

such that (F', E') is the concept of snakes.

Human beings think in protoconcepts

Given a formal context (G, M, I), the pairs (E, F) with $E \subseteq G$, $F \subseteq M$ and

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Basic elements of the algebraic theory have been investigated and also give some hints for their algorithmic treatment.

But given its fundamental importance to human thought, it seems to me that the theory of protoconcepts still has plenty of untapped potential.

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- how people collect and organize knowledge using preconcepts and protoconcepts,
- how they share such knowledge and thereby expand it,
- how formal contexts emerge from this in retrospect,
- and why ultimately the concept lattices are so informative.